Supply Chain Reliability and the Role of Individual Suppliers

AUTHORS

Simeon Hagspiel

EWI Working Paper, No 16/05

Original Version: April 2016
Updated Version: October 2016

Institute of Energy Economics at the University of Cologne (EWI)
www.ewi.uni-koeln.de

Institute of Energy Economics
at the University of Cologne (EWI)

Alte Wagenfabrik
Vogelsanger Str. 321a
50827 Köln
Germany

Tel.: +49 (0)221 277 29-100
Fax: +49 (0)221 277 29-400
www.ewi.uni-koeln.de

CORRESPONDING AUTHOR

Simeon Hagspiel
University of Cologne
Department of Economics | Institute of Energy Economics (EWI)
Tel: +49 (0)221 277 29-222
Fax: +49 (0)221 277 29-400
simeon.hagspiel@uni-koeln.de

ISSN: 1862-3808

Supply Chain Reliability and the Role of Interdependent Suppliers

Simeon Hagspiel[a,b,*]

[a] *University of Cologne, Department of Economics, Universitätsstrasse 22a, 50937 Cologne, Germany*
[b] *University of Cologne, Institute of Energy Economics, Vogelsanger Strasse 321a, 50827 Cologne, Germany*

Abstract

We study a one-period supply chain problem consisting of numerous suppliers delivering a homogenous good. Individual supply is uncertain and may exhibit dependencies with other suppliers as well as with the stochastic demand. Aiming at efficient strategies to manage supply chain reliability, we first derive a closed-form solution for an individual supplier's contribution, implicitly identifying changing returns to scale, gains of diversification, and non-additivity. We suggest two different approaches to explicitly account for the statistical properties of the problem: First, a comprehensive optimization model; and second, a straightforward solution for a local variant based on concepts from cooperative game theory. In a last step, we demonstrate practical relevance and applicability of our findings for the natural example of reliability in electricity supply systems facing increasing amounts of wind power.

Keywords: Supply chain reliability, Capacity uncertainty, Optimization, Cooperative game theory, Electric power industry

JEL: C44, C71, D47, L23, Q42, Q48

---

1. Introduction

Reliability is a key concern in virtually any supply chain. To manage and foster supply chain reliability, sourcing from multiple suppliers constitutes an effective and often applied practice. While the principle idea of risk diversification is quite simple, however, the organization and coordination of multiple suppliers to ensure a certain level of supply chain reliability at reasonable costs represents a challenging managerial task. This is particularly true when sourcing from external suppliers, but also holds when the supply chain is integrated. In this context, one major difficulty stems from the fact that individual suppliers are typically stochastic and interdependent. Consequently, their individual contributions to supply chain reliability do not only depend on their own characteristics, but also on all other suppliers in the chain.

An economic approach to establish a certain level of supply chain reliability consists in defining reliable supply as a service that can be provided by individual suppliers. The provision is then paid

---

Special thanks go to Joachim Bertsch, Clara Dewes, Felix Höffler and Ulrich Thonemann for valuable discussions and support. Funding by the German research society DFG through research grant HO5108/2-1 as well as through the DFG Zukunftskonzept (ZUK 81/1) is gratefully acknowledged.

*Email: simeon.hagspiel@ewi.uni-koeln.de

according to the individual supplier's contribution to supply chain reliability. An example for such explicit reliability-related payments can be found in electricity supply chains. Various systems – e.g., in the United States, France or the United Kingdom – have implemented so-called capacity mechanisms to ensure reliability of supply (for a good overview, see, e.g., Joskow (2008) or Cramton et al. (2013)). Key ingredient to all such mechanisms is the ex-ante determination of an individual supplier's contribution to supply chain reliability, commonly known as prequalification, which is used as a basis for subsequent payments. Despite the apparent relevance of this measure, however, it appears that existing approaches for its determination lack generality and consistency. Especially – as we will show in the course of this paper – they all fail to efficiently incorporate interdependencies between suppliers. As a matter of fact, we find that the concept of prequalification itself is imperfect in presence of interdependencies that only allow an ex-post derivation of consistent individual contributions. At the same time, efficient solutions are clearly needed to manage reliability in today's complex and interconnected supply chains.

Against this background, it is the goal of this paper to comprehensively investigate supply chain reliability and the role of interdependent suppliers. For this purpose, we consider a one-period supply chain problem consisting of multiple suppliers delivering a homogenous good.[1] Individual supply is uncertain and may exhibit dependencies with other suppliers as well as with the stochastic demand. Due to the stochastic nature of the problem, demand can be served at an intended reliability level, possibly including supply disruptions.[2] Aiming at efficient rules to organize and coordinate supply chain reliability, we are particularly interested in answering the following two questions: First, what is the contribution of individual suppliers to supply chain reliability? And second, how can these contributions be utilized in assessment and planning strategies to efficiently manage supply chain reliability?

To answer these questions, our paper contributes a comprehensive analytical framework as well as an empirical case study. In a first step, we derive a closed-form solution for an individual supplier's contribution, and implicitly identify changing returns to scale, gains of diversification, and non-additivity. In order to explicitly account for the statistical properties of the problem during the assessment and planning of supply chain reliability, we suggest two different approaches: First, a comprehensive optimization model; and second, a straightforward solution for a local variant to approximate the optimal solution based on concepts from cooperative game theory. These strategies may be employed to yield an efficient management of reliability in integrated or non-integrated supply chains.

Besides our analytical investigations, we apply our insights to the natural and relevant example

[1]The need to source a homogenous good from multiple suppliers may stem from capacity constraints or the requirement for risk diversification (e.g., Minner (2003) or Tang (2006)).

[2]Note that instead of *reliability*, one could also consider the generalized case of supply *quality*. However, for the sake of clarity, we will stick to the term reliability throughout the paper. Nevertheless, the concepts and results derived could also be applied to other dimensions of supply quality, such as time-to-respond, etc.

of reliability in power systems. Specifically, we will critically appraise existing mechanisms, and demonstrate practical applicability of our framework by means of a novel and pertinent empirical case study based on wind energy in the German power system. Moreover, we will show how our game-theoretic approach is able to improve today's flawed designs by providing a sophisticated prequalification of suppliers.

The rest of the paper is structured as follows: Section 2 reviews the related literature. Section 3 introduces our reliability model and solves for the contribution of individual suppliers, while the related statistical properties are depicted in Section 4. In Section 5, we investigate efficient strategies for the assessment and planning of supply chain reliability. The application of our findings to power systems is comprised in Section 6. Section 7 concludes.

2. Related literature

Our paper is closely related to the literature dealing with supply chain reliability and the problem of strategic sourcing of a homogenous good from multiple suppliers, facing either supply disruptions (i.e., a binomial distribution of uncertainty) or capacity uncertainty (i.e., an uncertain exogenous upper bound on the actual quantity supplied).[3]

Supply disruptions in a supply chain consisting of multiple suppliers have been studied by Parlar and Perry (1996), Gürler and Parlar (1997), Li et al. (2004) and Tomlin (2006), however, without considering stochastic dependencies among the suppliers. As a natural extension, later papers allow for dependencies between binomial supply disruptions (i.e., Babich et al. (2007), Wagner et al. (2009) and Li et al. (2010)).

Continuous distributions of the suppliers' uncertainties – as used in our paper – were presented by Dada et al. (2007) and Masih-Tehrani et al. (2011). The former paper proves that when selecting among a set of possible suppliers that are different in reliability and costs, cost generally takes precedence over reliability. While this result is derived under the assumption of supply distributions being independent, Masih-Tehrani et al. (2011) captures capacity uncertainty including multivariate dependencies, finding that the buyer's best strategy is risk diversification by choosing suppliers with independent distributions in order to avoid simultaneous supply disruptions.

In contrast to the above literature on supply chain reliability, our paper deviates in several important aspects. First, instead of analyzing the costs of supply chain reliability under exogenous prices,[4] we take a different perspective on the problem by endogenously determining the individual

[3]Note that the former is an extreme case of the latter. Also note that both are different from *yield uncertainty* which incorporates a dependency of the uncertainty on the order quantity. As we consider yield uncertainty as a different problem class, we do not review the related literature here. For a broader review of the literature dealing with supply chain risks, the reader is referred to Tang (2006) and Snyder et al. (2014).

[4]In the reviewed literature, suppliers' prices are either given as a parameter, or result from some supplier interaction (such as a Cournot game) without the buyer being able to have an influence.

supplier's value for supply chain reliability.[5] This allows us to disentangle the role of interdependent suppliers when managing large and complex supply chains. Second, we consider arbitrary and interdependent distributions of (un)availability and demand. And third, we present a novel solution based on concepts from cooperative game theory.

Our paper is also related to the more specific field of supply chain reliability in power systems, which has been investigated either from a technical or economic perspective. From a technical perspective, the goal has been to develop methodologies to assess the technical ability to provide reliability of supply (e.g., Garver (1966), Billinton (1970) or Billinton and Allan (1996)). The role of individual units for supply reliability, often referred to as capacity credit or capacity value, has also been discussed (for recent surveys, see, e.g., Amelin (2009) or Keane et al. (2011)). However, it appears that these analyses remain very technical and have never been transferred to a broader and generalized supply chain context. Moreover, they almost exclusively deal with the problem numerically. In contrast, our paper contributes a comprehensive and consistent analytical framework along with generalized implications for managing supply chain reliability.

Economically, power system reliability has been studied with respect to the (in)ability and potential failures of power markets to provide reliability as a market outcome (e.g., Joskow (2008) or Cramton et al. (2013)). However, even though various designs of capacity-related payoffs have been proposed, the role and implications of stochastic and interdependent suppliers have so far been disregarded. Our paper fills this gap by suggesting suitable approaches to incorporate those suppliers into reliability-related mechanisms in order to ensure economically efficient outcomes.

3. Reliability model

3.1. Supply chain reliability

We consider a one-period supply chain $\mathcal{S}$ consisting of numerous suppliers delivering a homogenous good. Suppliers are characterized by their joint stochastic availability of supply capacity C.[6] Demand D is also assumed stochastic. Due to considering only one period without additional backup (such as, e.g., inventory storage, emergency service, etc.), supply shortages occur whenever C is unable to cover D. Consequently, we define the risk of supply shortages in probabilistic terms as follows:[7]

[5] Technically, the difference stems from the fact that we determine the individual supplier's contribution based on an endogenous demand adjustment, while the literature's objective is to serve an exogenous (though, often stochastic) demand level.

[6] Here and in the following – unless indicated differently – capital letters are used for random variables.

[7] Note that we implicitly assume an inelastic demand with no reaction as capacity becomes scarce. If the good is marketed, this implies that market clearing cannot be guaranteed, e.g., because of the lack of real time pricing. Consequently, there is a risk of situations with all available capacities producing, but still being unable to fully serve demand – irrespective of the price level.

Definition 1. ***Risk of supply shortages:*** *Probability that supply chain $\mathcal{S}$, characterized by the stochastic availability of supply capacity C, is unable to cover stochastic demand D, i.e.:*

$$R^{\mathcal{S}} = \Pr(C \leq D) = \Pr(C - D \leq 0) = \Pr(X \leq 0) = F_X(0). \tag{1}$$

Correspondingly, supply chain reliability is defined as the complementary probability $1 - R^{\mathcal{S}}$. Note that in Equation (1), we have used F_X for the cumulative distribution function (cdf) of the excess capacity $X = C - D$. Alos note that stochastic dependencies between the random variables are so far unspecified. For instance, the overall availability of supply capacity C may result from multiple interdependent supply capacities of numerous individual suppliers. Furthermore, C may exhibit dependencies with the stochastic demand D.

3.2. The contribution of individual suppliers

We now investigate the contribution of an individual supplier with random production capacity to the reliability of supply chain $\mathcal{S}$. To this end, we add it to the existing system $\mathcal{S}$, and subsequently determine the risk of supply shortages of the new complemented supply chain $\mathcal{T}$ as:

$$R^{\mathcal{T}} = \Pr(C + Y \leq D) \tag{2}$$

Note that as Y is positive, $R^{\mathcal{T}} \leq R^{\mathcal{S}}$ always holds. With the goal to capture the contribution of Y to supply chain reliability, we follow the concept of incremental Value-at-Risk (VaR). I.e., we capture the change in risk exposure induced by the adjustment of X by Y while requiring the corresponding probability to remain at the original level (e.g., Tasche and Tibiletti (2003) or Jorion (2007)). In other words, we measure the additional demand that may be added to bring $R^{\mathcal{T}}$ back to the original risk level $R^{\mathcal{S}}$.[8,9] Formally, we define the contribution of an individual supplier as follows:

Definition 2. ***Contribution of an individual supplier to supply chain reliability:*** *Level of demand v by which D can be increased in order to maintain the original level of reliability, i.e.:*

$$\Pr(C + Y \leq D + v) = R^{\mathcal{S}} \tag{3}$$

Due to its complexity, Equation (3) is commonly solved for v numerically by means of iteration (see, e.g., Wang (2002) or Keane et al. (2011)). Advantages of the numerical solution include its straightforward implementability as well as the implicit coverage of statistical dependencies when using concurrent observations of demand and supply. However, numerical solutions – even

[8] Noticeably, this approach has been used in the context of power systems under the name of effective load carrying capability, which was originally developed by Garver (1966).

[9] As an alternative measure, one could also consider the change in reliability induced by adding supply capacity Y, i.e., $\Delta R = R^{\mathcal{S}} - R^{\mathcal{T}}$, however, without altering the principle results derived hereafter.

if conducted for a wide range of parameter constellations and application examples – do not allow for generalizations of the results obtained. Moreover, the numerical solution often entails a high computational burden.

In contrast, analytical solutions allow to readily calculate the desired results and provide further general insights. Nevertheless, only very few authors have engaged with the analytical analysis of Equation (3). In the literature related to portfolio risks, the incremental VaR is conveniently solved analytically up to a first order approximation (e.g., Tasche and Tibiletti (2003) or Jorion (2007)). In contrast, Dragoon and Dvortsov (2006) propose the z-method, considering higher order terms and the special case of a normally distributed capacity shortage over demand and independence with the individual supplier (i.e., $X \sim \mathcal{N}$ and $X \perp Y$). Extending this approach, Zachary and Dent (2011) proof a closed-form solution with an arbitrary distribution of X and independent X and Y. Even though the latter paper discusses the natural extension to the case of dependent distributions, the formal proof is not included. Hence, in the following proposition we present the generalized solution of Equation (3) for v, with arbitrary dependence between the distributions of X and Y, and $\sigma_{(\cdot)}$ being their standard deviation.

Proposition 1. *For $\sigma_Y \ll \sigma_X$, the contribution v of an individual capacity Y to supply chain reliability is approximated by*

$$v = \mu_{Y|X\approx 0} - \frac{\sigma^2_{Y|X\approx 0}}{2} \frac{f'_X(0)}{f_X(0)}. \tag{4}$$

Proof. From Equations (3) and (1) it follows that the equation to be solved for v is

$$\Pr(C + Y \leq D + v) = \Pr(X + Y \leq v) = \Pr(X \leq 0) = F_X(0). \tag{5}$$

The cumulative distribution function $\Pr(X + Y \leq v)$ can be expressed as integrals over the joint probability density function $f_{X,Y}$, with X, Y being arbitrary dependent distributions. Reformulating using conditional distributions and integrating over x, we obtain

$$\Pr(X + Y \leq v) = \int_{-\infty}^{\infty} \int_{-\infty}^{x=v-y} f_{X,Y}(x, y) dx dy \tag{6}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{x=v-y} f_{Y|X}(y|x) f_X(x) dx dy \tag{7}$$

$$= \int_{-\infty}^{\infty} f_{Y|X=v-y}(y|X = v - y) F_X(v - y) dy. \tag{8}$$

Instead of continuing with the explicit form of the cumulative distribution function F_X, we approximate it via Taylor expansion around the critical point $v - y = 0$ up to the second order polynomial degree, i.e.,

$$F_X(v - y) = F_X(0) + f_X(0)(v - y) + \frac{f'_X(0)}{2}(v - y)^2. \tag{9}$$

Noticeably, in the above equation we have induced and accepted an approximation error of $o((v-y)^2)$, which occurs if derivatives of order two or higher are non-zero.

Next, note that if $\sigma_Y \ll \sigma_X$, it follows that $\Pr(X+Y \leq v) \approx \Pr(X+\mu_Y \leq v)$ and hence, that $v \approx \mu_Y$ in Equation (5). We now insert (8) and (9) in (5), and reformulate using the concept of conditional expectations:

$$\int_{-\infty}^{\infty} f_{Y|X=v-y}(y|X \leq v-y) \left[F_X(0) + f_X(0)(v-y) + \frac{f'_X(0)}{2}(v-y)^2\right] dy \tag{10}$$

$$= F_X(0) + f_X(0)\mathbb{E}\left[(v-Y)|X \approx 0\right] + \frac{f'_X(0)}{2}\mathbb{E}\left[(v-Y)^2|X \approx 0\right] = F_X(0), \tag{11}$$

which can be simplified using standard deviation σ as well as expected values μ to

$$f_X(0)(v-\mu_{Y|X\approx 0}) + \frac{f'_X(0)}{2}\left((v-\mu_{Y|X\approx 0})^2 + \sigma^2_{Y|X\approx 0}\right) = 0. \tag{12}$$

Equation (12) represents a quadratic equation in $(v-\mu_{Y|X\approx 0})$ that can readily be solved for v based on the assumption of small σ_Y (so that the error of order $\mathcal{O}(\sigma^4_{Y|X\approx 0})$ is small), such that Equation (4) follows. □

From Equation (4), we observe that two terms including two different statistical features of the individual supplier are decisive for its contribution to supply chain reliability: its average availability μ on the one hand, and its standard deviation σ on the other, both at times of scarce capacity, i.e. for $X \approx 0$. Specifically, higher average availability at times of critical capacity directly contributes to reliability. In contrast, the effect of the standard deviation getting larger may be either positive or negative for reliability, depending on the sign of $f'_X(0)$ (i.e., the convexity of the cdf).[10,11] It should typically hold true that high levels of reliability are required from supply chains, such that the critical point $X = 0$ is located at the left hand side of the distribution where the probability for insufficient capacity is low and increasing in x, which yields $f'(x) > 0$. Then, the individual supplier's contribution to reliability decreases in its standard deviation. Somewhat counterintuitive, however, the individual supplier's contribution to reliability may also *benefit* from a high standard deviation. This is the case for $f'(x) < 0$, resulting in a situation where positive deviations weight more than negative ones. This could hold true and represent an interesting feature, e.g., for supply chains that are still at an early stage of development while already facing high levels of demand, or for well-established systems that underwent a sudden and substantial

[10]Note that the density function $f(x)$ is always positive by definition.

[11]The effect of the standard deviation essentially stems from the difference in impact from positive and negative deviations from the average availability. If positive and negative deviations had an equal impact and would outweigh each other, the *expected* overall contribution would not change. However, the impact of the standard deviation on the reliability contribution increases with the absolute level of $f'(x) = F''(x)$, i.e., with the level of convexity of the cumulative distribution function $F(x)$. It also increases with $f(x) = F'(x)$ (i.e., the slope of the cdf) decreasing, as the difference in impact from positive and negative deviations then becomes more important.

increase in demand.

Two further points are worth mentioning. First, the contribution of an individual supplier to supply chain reliability may – instead of absolute numbers – be reported relative to its maximum available capacity $\overline{y} = \max Y$, i.e., as a fraction $\tilde{v} = v/\overline{y}$, with $0 \leq \tilde{v} \leq 1$. Second, Equation (4) can readily be extended to multiple (n) suppliers contributing jointly to supply chain reliability. In this case, define Y as the sum of the individual suppliers S_i, i.e., $Y = \sum_{i=1}^{n} Y_i$. Denoting with N the set of all n suppliers in the system and contributing jointly to generation adequacy, we will write $v(N)$ for their joint contribution. To determine the joint contribution of one or a few units only, we will use set $S \subseteq N$, and denote the corresponding contribution by $v(S)$. The joint contribution of multiple units (or, in other words, a coalition of units) will become important for the subsequent analysis.

4. Statistical properties

In the following, we present three corollaries describing essential statistical properties of the reliability contribution(s) v, following from the explicit formulation as provided in Proposition 1. Moreover, we depict and discuss important implications for the efficient management of supply chain reliability involving multiple suppliers delivering a homogenous good.

4.1. Changing returns to scale

Corollary 1. *The contribution of an individual supplier is subject to changing returns to scale if* $\sigma_{Y|X\approx 0} > 0$ *and* $f'_X(0) \neq 0$.

Returns to scale can easily be shown by inserting a scaled random production capacity aY in Equation (4), and using the fact that both, mean and standard deviation scale directly with the scaling factor of the random variable. It follows that

$$v(S_a) = \mu_{aY|X\approx 0} - \frac{\sigma^2_{aY|X\approx 0}}{2}\frac{f'_X(0)}{f_X(0)} \tag{13}$$

$$= a\mu_{Y|X\approx 0} - \frac{a^2\sigma^2_{Y|X\approx 0}}{2}\frac{f'_X(0)}{f_X(0)}. \tag{14}$$

Whereas the first term on the right hand side increases linearly with a, the second term decreases with higher order a^2 as long as $\sigma_{Y|X\approx 0} > 0$ and $f'(x) > 0$. Under these conditions, it holds that $v(S_a) < av(S)$, i.e., decreasing returns to scale result when considering an increasing amount of capacity with equal availability Y. In contrast, $\sigma_{Y|X\approx 0} > 0$ and $f'(x) < 0$ yields increasing returns to scale.

From a management perspective, Corollary 1 is of particular relevance for a supply chain that shall increasingly rely on one particular supplier, e.g., due to relative cost advantages. For illustration, imagine a power system that aims at replacing an increasing number of fossil power plants with wind power capacities while keeping its original reliability level (suppose here a reliable power system with $f'_X(0) > 0$, and variable wind power resources with $\sigma_{Y|X\approx 0} > 0$). Corollary 1 implies that with each additional unit of wind power installed, a decreasing amount of fossil power can be safely removed from the system.

4.2. Gains of diversification

Corollary 2. *Similar average availability and low (high) covariance in case of $f'_X(0) > 0$ ($f'_X(0) < 0$) entails gains of diversification for the contribution of individual suppliers.*

For illustration, let us assess the contribution of a supplier S_1 with some capacity Y_1 to supply chain reliability. Assume that some share $\alpha \in [0, 1]$ of capacity Y_1 can be sourced from an alternative supplier S_2. Depending on the choice of α, the joint capacity becomes $Y_{12} = (1 - \alpha)Y_1 + \alpha Y_2$. Denote with $w(\cdot)$ the functional dependence of the joint contribution on α, i.e. define $w(\alpha) = v(S_{1,1-\alpha} \cup S_{2,\alpha})$. Then, the joint contribution of this portfolio to supply chain reliability writes as

$$w(\alpha) = \mu_{Y_{12}|X\approx 0} - \frac{\sigma^2_{Y_{12}|X\approx 0}}{2} \frac{f'_X(0)}{f_X(0)} \tag{15}$$

$$= (1 - \alpha)\mu_{Y_1|X\approx 0} + \alpha\mu_{Y_2|X\approx 0} - \left((1 - \alpha)^2\sigma^2_{Y_1|X\approx 0} + \alpha^2\sigma^2_{Y_2|X\approx 0} + 2\alpha(1 - \alpha)\sigma_{Y_1,Y_2|X\approx 0}\right) \frac{f'_X(0)}{2f_X(0)}. \tag{16}$$

To identify gains of diversification, we need to check the derivative of w with respect to α in the region of $\alpha = 0$, i.e.,

$$\left.\frac{\partial}{\partial\alpha} w(\alpha)\right|_{\alpha=0} = -\mu_{Y_1|X\approx 0} + \mu_{Y_2|X\approx 0} + \left((1 - \alpha)\sigma^2_{Y_1|X\approx 0} - \alpha\sigma^2_{Y_2|X\approx 0} - (1 - 2\alpha)\sigma_{Y_1,Y_2|X\approx 0}\right) \left.\frac{f'_X(0)}{f_X(0)}\right|_{\alpha=0} \tag{17}$$

$$= -\mu_{Y_1|X\approx 0} + \mu_{Y_2|X\approx 0} + \left(\sigma^2_{Y_1|X\approx 0} - \sigma_{Y_1,Y_2|X\approx 0}\right) \frac{f'_X(0)}{f_X(0)}. \tag{18}$$

From the derivative evaluated at $\alpha = 0$, we observe that it is positive (and hence, $w(\alpha)$ increasing) as long as the average availability of supplier S_2 is similar to that of supplier S_1, and their covariance is either *not* particularly strong in case of $f'_X(0) > 0$, or particularly strong otherwise. Under those conditions, the joint contribution is subject to gains of diversification.

The meaning of gains from diversification, as identified in Corollary 2, is quite intuitive, and essentially formalizes the "don't put all eggs into one basket" wisdom. Hence, a diversified portfolio with multiple suppliers is often better able to reliably supply a certain load level. However, this only holds true as long as the expected yield is similar and they are not highly correlated (for the typically more relevant case of $f'_X(0) > 0$).[12]

[12]Note that in finance, the concept of risk spreading is well-known from Markowitz' portfolio theory that shows

4.3. Non-additivity

Corollary 3. *As long as $f'_X(0) \neq 0$, contributions of individual suppliers are non-additive if they are stochastically dependent.*

For illustration, let us again consider a supply chain comprising two suppliers S_1 and S_2 with (possibly dependent) production capacities Y_1 and Y_2, respectively. The joint contribution of S_1 and S_2 to supply chain reliability writes as

$$v(S_1 \cup S_2) = \mu_{Y_{12}|X} - \frac{\sigma^2_{Y_{12}|X}}{2} \frac{f'_X(0)}{f_X(0)} \tag{19}$$

$$= \mu_{Y_1|X\approx 0} + \mu_{Y_2|X\approx 0} - \left(\sigma^2_{Y_1|X\approx 0} + \sigma^2_{Y_2|X\approx 0} + 2\sigma_{Y_1,Y_2|X\approx 0}\right) \frac{f'_X(0)}{2f_X(0)}. \tag{20}$$

In contrast, if these suppliers were to be assessed independently, we can simply apply Equation (4) to each of them to get

$$v(S_1) = \mu_{Y_1|X\approx 0} - \frac{\sigma^2_{Y_1|X\approx 0}}{2} \frac{f'_X(0)}{f_X(0)} \tag{21}$$

$$v(S_2) = \mu_{Y_2|X\approx 0} - \frac{\sigma^2_{Y_2|X\approx 0}}{2} \frac{f'_X(0)}{f_X(0)}. \tag{22}$$

From the above equations, we see that as long as $\sigma_{Y_1,Y_2|X\approx 0} \neq 0$, it follows that $v(S_1 \cup S_2) \neq v(S_1) + v(S_2)$. Note that generalization to n suppliers in the system yields $\sum_{i=1}^{n} v(\{i\}) \neq v(N)$ as long as $\sigma_{Y_j,Y_k|X\approx 0} \neq 0$ for any pair j, k.

Let us discuss three important implications following from Corollary 3: First, note that even though individual contributions can technically be determined, they are inconclusive under any type of (positive or negative) dependence between the suppliers' availabilities. This is due to the fact that the relevant figure for supply chain reliability is the joint contribution of all suppliers which incorporates externalities that cannot be reflected in individual assessments. We can infer that a comprehensive approach is imperative to assess and coordinate supply chain reliability in order to reach a consistent representation of all interdependent suppliers.

Second, if the suppliers were to coordinate themselves (e.g., in case the supply chain is non-integrated), they would have no incentive to team up for joint contributions if they are positively

similar characteristics when valuing properties of multi-asset portfolios (assets are here the equivalent to our suppliers). However, while the interest in a Markowitz portfolio lies in finding an efficient *tradeoff* between risk and expected returns, we are here interested in reliability contributions which are, as found in Equation (4), dependent on *both* variance *and* expectation (together with additional characteristics captured by f_X which are not occurring in Markowitz). Also note that one of the main criticisms with respect to Markowitz' portfolio theory, namely the linear dependence assumption between the joint distributions, also applies to our case here. In fact, non-linear dependencies can indeed be relevant for supply chain risks, as shown, e.g., by Wagner et al. (2009), Masih-Tehrani et al. (2011) or Elberg and Hagspiel (2015). While this would be an interesting extension of our analysis, it would go beyond the scope of this paper and is left for future research.

related. Technically, positive dependencies imply subadditivity (or, in other words, a negative externality) and hence, an empty core if the situation is characterized as a game. This would substantially complicate the implementation of a competitively stable result (Telser (1994)). Even though we will reconsider this issue in the next section, we already note that a complete analysis of coordination mechanisms for non-integrated supply chains would be beyond the scope of this paper and is left for future research.

Third, we observe that the value of an individual supplier for reliability depends on the other suppliers in the chain. Hence, if the effective set of interdependent suppliers is unknown ex-ante (e.g., if the system is to emerge from some "unforeseeable" development, such as an auction), it is impossible to predict the individual suppliers' contribution to reliability (or, in other words, to conduct a prequalification). This is due to the fact that positive (negative) dependencies cause negative (positive) externalities that cannot be internalized as long as their size is unknown. Note that this problem would not be present (and hence, a predetermination possible) if all suppliers were independent (and hence, externalities non-existent).

5. Assessment and planning strategies

Based on the previous results and findings, we now investigate suitable strategies for the assessment and planning of reliability in a multiple sourcing problem with possibly interdependent suppliers. Especially, we will deal with the symptomatic challenge of efficiently expanding an existing pool of suppliers to reach a more reliable supply, or its reduction to save the related costs. We will first formulate a model to optimize supply chain reliability in an integrated and comprehensive way, and then switch our focus to a local variant of the problem with a straightforward solution based on game-theoretic foundations.

5.1. Comprehensive optimization

Let us assume there is a set of potential suppliers M, each characterized by the supply characteristics $\mu_{Y_i|X\approx 0}$ and $\sigma_{Y_i|X\approx 0}$ as well as the related supplier's costs c_i. The managerial task consists of deciding which or, alternatively, to which extent to rely on a set of players $N \subseteq M$ that shall be contracted for a reliable delivery.

Unfortunately, in order to make an efficient choice, Corollary 3 prohibits an easily applicable ranking of the potential suppliers according to their individually determined reliability contributions (weighted with their corresponding costs). In contrast, we need a comprehensive model that builds upon the supply and cost characteristics of all suppliers to find an optimal portfolio. As such, externalities form statistical dependencies can consistently be represented and internalized in the (efficient) solution.

As above, denote with $w(\cdot)$ the functional dependency of the selected portfolio's contribution to reliability, i.e., $w(x_i) = v(\bigcup_{i\in M} S_{i,x_i})$, where each player's capacity has been multiplied with the choice variable x_i (i.e., the joint capacity is $Y = \sum_{i=1}^{n} x_i Y_i$). Assuming that the objective is to maximize the suppliers' reliability under a given budget-constraint B, the integrated planning problem can be formalized as follows:

$$\max_{x_{i\in M}} \quad w(x_i) \tag{23}$$

$$\text{s.t.} \quad \sum_{i\in M} x_i c_i \leq B. \tag{24}$$

Note that we have so far not specified the nature of our choice variables x_i. We will in the following discuss two distinct cases: First, when x_i is binary, and second, when x_i is positive continuous. Both cases may be meaningful for management purposes.

5.1.1. Binary variables

If x_i is binary, i.e., if we can choose whether or not to take a supplier into our portfolio, we are facing a well-known combinatorial optimization problem: a quadratic knapsack problem (e.g., Gallo et al. (1980)). More specifically – unless $\sigma_{Y_i,Y_j|X\approx 0} \geq 0$ for all pairs i, j and $f'_X(0) \leq 0$ – we have a quadratic knapsack problem with positive and negative profits. As shown in Rader and Woeginger (2002), this class of problems does not have any polynomial time approximation algorithm with fixed approximation ratio unless $\mathcal{P} = \mathcal{NP}$. However, there are numerous papers devoted to upper bounds, size reduction, heuristics, approximation techniques and exact algorithms. The interested reader is referred to a good survey presented by Pisinger (2007) for further details about quadratic knapsack problems.

5.1.2. Positive continuous variables

If $x_i \in \mathbb{R}_{\geq 0}$, problem (23)-(23) represents a nonlinear optimization problem. Specifically, we have a quadratic objective function and linear constraints for which the solution strongly depends on the sign of $f'_X(0)$.

Proposition 2. *If $f'_X(0) \geq 0$, problem* (23)-(23) *can be solved efficiently to global optimality.*

Proof. The joint contribution of a set of suppliers with capacities $x_i Y_i$ can be rewritten in matrix notation as:

$$w(x_i) = \boldsymbol{x}^T \boldsymbol{\mu} - \boldsymbol{x}^T \Sigma \boldsymbol{x} \frac{f'_X(0)}{2 f_X(0)}, \tag{25}$$

where $\boldsymbol{x}$ is the vector containing all x_i and $\boldsymbol{\mu}$ containing all μ_{Y_i}, with $i \in M$. Noticeably, Σ is exactly the covariance matrix of the suppliers. As the covariance matrix is positive-semidefinite by definition, the Hessian of $w(x_i)$ is negative-semidefinite as long as $f'_X(0) \geq 0$. If so, $w(x_i)$ is a concave function and hence, problem (23)-(23) represents a concave quadratic maximization with linear constraints which can be solved efficiently to global optimality in polynomial time using standard techniques (e.g., Bazaraa et al. (2006)). □

If $f'_X(0) \leq 0$, problem (23)-(23) becomes a convex quadratic maximization, and the application of local optimization can no longer guarantee that the solution is a global optimum. However, it is well known that all solutions of such a problem must lie at some vertex of the feasible region, with the challenge to identify whether the solution obtained is a global or only a local optimum. Known approaches build on cutting plane methods, concave envelopes, or reduction techniques. Instead of discussing additional properties of the problem, however, we refer the interested reader to Floudas and Visweswaran (1995) for further details.

We conclude our discussion about the comprehensive optimization of supply chain reliability with an illustrative example.

Example. Consider problem (23)-(23) for the case of two suppliers, i.e., $i \in 1, 2$, and continuous decision variables $x_i \in \mathbb{R}_{\geq 0}$. Furthermore, assume that costs c_i are equal for both suppliers, i.e. $c_1 = c_2 = c$. As reliability is increasing in x_i, it follows that the budget constraint will be binding, indicating that only a limited capacity $K = x_1 + x_2 = B/c$ can be built. Consequently, we only need to derive the best diversification strategy to distribute K among the two suppliers S_1 and S_2. I.e., we need to find $\alpha \in [0, 1]$ such that the production capacities $(1 - \alpha)Y_1$ and αY_2 jointly yield the best overall level of reliability. Hence, the optimization takes the following form:

$$\max_{\alpha} \quad w(\alpha) = (1-\alpha)\mu_{Y_1|X\approx 0} + \alpha\mu_{Y_2|X\approx 0} - \Big((1-\alpha)^2\sigma^2_{Y_1|X\approx 0} + \alpha^2\sigma^2_{Y_2|X\approx 0} + 2\alpha(1-\alpha)\sigma_{Y_1,Y_2|X\approx 0}\Big)\frac{f'_X(0)}{2f_X(0)}. \tag{26}$$

We differentiate (26) with respect to α which – after a few calculations – yields:

$$\alpha^* = \frac{\left(\mu_{Y_2|X\approx 0} - \mu_{Y_1|X\approx 0}\right)\frac{f_X(0)}{f'_X(0)} + \sigma^2_{Y_1|X\approx 0} - \sigma_{Y_1,Y_2|X\approx 0}}{\sigma^2_{Y_1|X\approx 0} + \sigma^2_{Y_2|X\approx 0} - 2\sigma_{Y_1,Y_2|X\approx 0}}. \tag{27}$$

As expected, increasing weight is placed on S_2 when $\mu_{Y_2|X\approx 0}$ (given that $f'_X(0) > 0$) or $\sigma_{Y_1|X\approx 0}$ is increasing (or vice versa for their respective counterparts). The effect is amplified by an increasing covariance. Note that if the two suppliers are identical (i.e., when $\mu_{Y_1|X\approx 0} = \mu_{Y_2|X\approx 0}$ and $\sigma_{Y_1|X\approx 0} = \sigma_{Y_2|X\approx 0}$), then $\alpha^* = 1/2$ in order to optimally deploy changing returns to scale based and gains from diversification.

5.2. A local variant

Let us now switch our focus to a local variant of problem (23)-(23), where we presuppose that we are dealing with a predefined set of suppliers N. For instance, this could be an existing portfolio of suppliers, or a readymade plan for a future set of suppliers. Aiming at a comparison of individual suppliers against each other or against the chain's value of reliability (e.g., in order to decide on possible cutbacks in the chain), the managerial objective is to determine each supplier's contribution to reliability. Note that by fixing the set of suppliers we are able to predetermine and internalize the externalities stemming from statistical interdependencies between the suppliers. Nevertheless, even though the joint value is now easy to derive, it remains challenging to allocate an individual supplier's value in a suitable and meaningful way. We will approach and solve this problem based on easily applicable game-theoretic foundations.

To this end, characterize the (joint) value of one or multiple suppliers for supply chain reliability as the output of a coalitional game (N, v) with transferable utility, where N is a finite set of units in the system, and v a characteristic function. $v(\cdot)$ measures the (joint) contribution of a nonempty coalition of suppliers $S \subseteq N$ as defined implicitly in Equation (3), or explicitly in Equation (4). Note that $v(S) \in \mathbb{R}^s$, i.e., for every coalition S, a corresponding contribution can be determined. A solution concept for this coalitional game is a payoff vector $\Phi \in \mathbb{R}^N$ allocating the joint value $v(N)$ to the coalition members.

While there would be an infinite number of allocation rules to split the joint contribution and allocate it to individual suppliers (e.g., the joint contribution could simply be allocated to one single supplier, or be split into equal parts), it is important to note that the specific choice may feed back to the supply chain performance and should hence be chosen with care. Specifically, the rule should reflect individual marginal contributions to reliability in an efficient way.

The efficiency property simply requires that the sum of all allocations adds up to the worth of all suppliers contributing jointly to reliability, i.e., that $\sum_i \Phi_i(N, v) = v(N)$. As we have seen in Corollary 3, this requires to internalize externalities occurring due to statistical interdependencies.

Following Hart and Mas-Colell (1989), we define the marginal contribution of a supplier to reliability as $D^i P(N, v) = P(N, v) - P(N \setminus \{i\}, v)$, where $P(N, v)$ is a single real number associated to the game (N, v), and $P(\emptyset, v) = 0$. P is called a potential function if it satisfies the efficiency condition $\sum_i D^i P(N, v) = v(N)$.

Based on the above formalizations, the allocation problem can be solved by means of a theorem presented in Hart and Mas-Colell (1989).

Theorem 1. ***(Hart and Mas-Colell (1989))***. *There exists a unique potential function P. Moreover, the corresponding vector of marginal contributions $D_i P(N, v)_{i \in N}$ coincides with the Shapley value of the game (N, v), given by:*

$$\Phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S)). \tag{28}$$

Noticeably, the solution based on the Shapley value not only solves the allocation problem in a unique way, but is also normatively fair in the sense that it (uniquely) fulfills the following additional axioms: Symmetry, Linearity, Dummy, and Consistency (Shapley (1953) and Hart and Mas-Colell (1989)).

With its properties, we can use the Shapley value to allocate the suppliers' joint contribution, and to rank the individual suppliers' marginal contribution to reliability in an efficient way. Thus, it may serve as a powerful tool for planning and controlling purposes, e.g. in order to quantify and compare the reliability of individual suppliers against each other. Moreover, weighted with its corresponding costs, a supplier's contribution may also be compared against the chain's value of reliability VoR (in the context of electricity, this is often referred to as the value of lost load). The necessary condition to keep supplier i in the chain would then write as

$$\frac{c_i}{\Phi_i} \leq VoR. \tag{29}$$

Our allocation rule could also serve as a basis for a more comprehensive expansion planning: Due to the fact that the Shapley value corresponds to the (discrete) gradient of the game, it provides the direction of the greatest increase in reliability. Hence, as long as adjustments are sufficiently small, increasing capacities according to this direction is the optimal expansion strategy.[13]

Solving the reliability assessment and planning problem based on the Shapley value has an addition appeal in that it can also be used as a signal and rule of valuation for external suppliers in the context of non-integrated supply chains, thus reaching a normatively fair valuation and triggering efficient investment incentives. In fact, incentivized by payments according to the Shapley value, a supplier will find it optimal to align her/his investments according to the supply chain optimal solution as this provides the largest payoff. For illustration, reconsider the example from Section 5.1.

Example (continued). Investment incentives induced by the above allocation rule (Equation (28)) may be investigated by means of biform games, studying the impact of surplus division on non-cooperative investment decisions (Brandenburger and Stuart (2007)). Investments take place in a stage prior to surplus allocation, such that the suppliers' profits are maximized.

Comparable to the first part of the example dealing with the integrated planning of supply chain reliability, we now consider the case of a price-taking external investor who wants to build supply capacity $K \in [0, \inf)$, and needs to decide on the share $\beta \in [0, 1]$ (respectively $1 - \beta$) to be invested in supplier S_2 (S_1). Under the assumption that revenues are generated from reliability-related payoffs according to the Shapley value, the investor's profit function becomes

$$\Pi = p((1 - \beta)\Phi_1 + \beta\Phi_2) - Kc, \tag{30}$$

with p the (constant) price paid per reliable capacity (e.g., determined through an assessment of the chain's value of reliability, as previously discussed). For the case of two suppliers, the Shapley

[13] If adjustments are large, the gradient would need to be recalculated, thus corresponding to the gradient descent iterative optimization algorithm. In fact, this would be one of the available algorithms to solve the optimization problem (23)-(23) from above.

values Φ_1, Φ_2 can easily be calculated as

$$\Phi_1 = \frac{1}{2}v(S_1) + \frac{1}{2}(v(S_1 \cup S_2) - v(S_2)) \tag{31}$$

$$\Phi_2 = \frac{1}{2}v(S_2) + \frac{1}{2}(v(S_1 \cup S_2) - v(S_1)). \tag{32}$$

Inserted in Equation (30), the optimal weight for the distribution of the investment can be derived from the first-order condition $\frac{\partial \Pi}{\partial \beta} = 0$, which – after a few calculations – yields β^*. We find that $\beta^* = \alpha^*$, i.e., equality of the investor's optimal choice and the optimal integrated planning benchmark.

Two further remarks are worthwhile: First, note that it always needs a certain price level to incentivize investments. Specifically, it would be individually rational to invest as long as $\frac{p}{c} > \frac{K}{v((1-\beta)S_1 \cup \beta S_2)}$. Changing prices would not affect the distribution of investment due to the linearity property of the Shapley value. Second, also note that reliability-related payoffs could be complemented with other payoffs, e.g. for expected production volumes.

6. Application to power systems

Power systems can be seen as a natural example of our previously investigated problem, due to the fact that a highly homogenous good (i.e., electricity) is sourced from a large number of capacity constrained suppliers (i.e., power plants). Moreover, reliability is a key concern as uninterrupted electricity supply is of high economic value, and blackouts can cause tremendous losses. For instance, in the early 2000s insufficient supply capacities caused a series of blackouts in the Californian power system affecting several hundred thousand customers. The State of California was forced to initiate short-term countermeasures to alleviate the crisis, amounting to an estimated 40 bn.$ in additional energy costs from 2001 to 2003 (Weare (2003)). The economy was estimated to slow down by 0.7-1.5%, entailing an increase in unemployment by 1.1% (Cambridge Energy Research Associates (2001)). The severity of these events has triggered a worldwide debate about reliability in power systems as well as the development of supporting mechanisms (i.e., so-called capacity mechanisms). Noticeably, the discussion has recently been regaining momentum due to the large-scale deployment of variable renewable energies (such as wind or solar power) whose impact on power system reliability is considered a *crucial issue of common interest* (Council of European Energy Regulators (2014)).

Against this background, this section has two objectives: First, to use our generated insights to critically appraise current mechanisms supporting reliability in power system. And second, to demonstrate applicability and relevance of our previous investigations in an empirical case study for wind power in Germany.

6.1. Critical appraisal of current mechanisms

Despite a broad consensus on the importance of reliability in the power sector, it appears that existing mechanisms clearly lack generality and consistency, especially when it comes to the role of variable and interdependent renewable energies. This is reflected in a variety of fundamentally different approaches, clearly revealing that there is no best practice (let alone an optimal benchmark) for how to deal with the reliability of such resources. Moreover, it appears that (negative) externalities from statistical interdependencies have simply been disregarded in current mechanisms.

Based on the findings presented in the previous part of the paper, let us reflect upon three representative mechanism designs. First, consider the case of Spain. They exclude variable resources (i.e., wind and solar power) by allocating zero reliable capacity. This clearly induces a situation of static inefficiency as soon as there is any positive contribution – which is likely to be the case as shown in numerous studies (e.g., Keane et al. (2011) or our own empirical analysis presented hereafter). Note that by allocating a value of zero, the Spanish design also fails to incentivize investments into suppliers that are beneficial for reliability, hence also inducing an additional long-term inefficiency.

As a second example, take the capacity auction of New England (PJM market). They attempt to prequalify suppliers individually in an ex-ante procedure. This clearly contradicts the non-additivity we have found in Corollary 3. Moreover, while PJM allows suppliers using intermittent resources to cooperate and bid jointly, there will typically be disincentives to do so due to positive dependencies in the availability of generation capacity. These shortcomings will necessarily entail an inefficient outcome.

Third, drawbacks can also be found in the UK capacity mechanism which allocates an average payoff to each unit in the same technology-class (e.g., coal, gas, wind, etc.). While the approach ensures static efficiency by complying with the true joint contribution, it lacks the provision of spatially diversified investment incentives. In addition, the approach may face opposition from underrated suppliers whose individual contribution to reliability lies well above the average level.

A last and most striking design fault is present in *all* existing approaches. They all build on the concept of prequalification, i.e., an ex-ante assessment of all units *potentially* participating in the chain yielding a single number to reflect its contribution to reliability of supply. Subsequently, a mechanism (e.g., an auction) selects the *effective* subset that shall be realized. In the presence of interdependencies, this is simply not possible. As it was discussed in the context of the Nonadditivity-Corollary 3, the value of an individual supplier for reliability depends on the effectively realized set of suppliers in the supply chain due to the externalities that can only be known ex-post. In a changing world with highly interdependent resources for electricity generation, future mechanisms will hence need to find solutions in a comprehensive model based on the full statistical characteristics of the capacity availability as well as the costs of each supplier, as it was shown in Section 5.1. This should clearly be the medium- and long-term strategy to organize and coordinate supply chain reliability in the electricity sector.

In a short-term perspective, however, the solutions presented in Section 5.2 can be a suitable mean to improve the efficiency of mechanisms while maintaining their principle design features, i.e., an ex-ante assessment (i.e., prequalification) of all units with a single number and a subsequent selection of an effectively delivering subset, e.g., via a capacity auction. In fact, it can be argued that power systems are typically quite mature, already comprising an almost sufficient amount of generation capacity. In such a context, the setting would at least be close to the one considered in Section 5.2. Hence, calculating the Shapley value for the existing suppliers would yield their (true) contribution to system reliability, which they could be prequalified with previous to a capacity auction. In addition, (comparatively few) new suppliers interested in entering the market would be prequalified with the same (relative) value as an already existing supplier that is located nearby and hence, shows similar capacity availability as the new entrant. As long as changes are sufficiently small, this yields the supply chain optimal extension. For illustration, we will provide an empirical example for this approach in the next section.

6.2. Empirical case study

Based on real-world data for wind power in Germany, we shall now develop an empirical case study to demonstrate applicability and relevance of the concepts developed in the previous sections and to empirically confirm our analytical results. Specifically, we shall quantify the reliability of the supply system as well as the joint contribution of wind power, and then derive the individual allocations according to the Shapley value.

Note that the case of wind power in Germany appears to be particularly relevant: With 34.02 out of 188.68 GW production capacity being installed, wind power plays a major role in Germany's generation portfolio, while it is a declared goal to integrate the technology into general market structures. Moreover, power system reliability as well as possible regulatory interventions, such as capacity mechanisms, have been heavily discussed for several years.

6.2.1. Estimation procedure

Recall Equation (3) that needs to be solved for v in order to obtain the contribution of individual suppliers. We assume random variable C, i.e., the availability of the power supply fleet apart from wind power, to be independent from wind power Y and demand D, and that its distribution can be determined through convolution of the suppliers' outage probabilities (see below). In contrast, the joint distribution of Y and D is estimated from simultaneous historical observations. As we only have one observation per instant in time t, we need to extend Equation (3) to multiple time periods, such that random variables Y, D may be replaced by corresponding observations y_t, d_t:[14]

$$\sum_{t=1}^{T} \Pr(C + y_t \leq d_t + v) = \sum_{t=1}^{T} \Pr(C \leq d_t) \tag{33}$$

[14] The validity and consistency of the result obtained from this reformulation may be justified by the central limit theorem (Zachary and Dent (2011)).

Note that summing up the probabilities over time yields the expected value during the considered number of hours T. This measure is often applied to formulate or benchmark reliability levels. For instance, a 1-day-in-10-years criterion has often been used as a benchmark or target value, both in the academic literature (e.g., see Keane et al. (2011)) as well as in practice (e.g., by the Midcontinent ISO or the ISO New England).[15]

6.2.2. Data

The necessary data can be classified into three main areas: First, detailed information is needed about installed capacities and availability factors of dispatchable power suppliers apart from wind (C in Equation (33)). Second, the analysis requires high-resolution data on wind power capacities as well as their infeed profiles (y_t). Third, load levels with the same temporal resolution and regional coverage are needed to perform the calculations (d_t). Descriptions of the data along with some preparatory calculations can be found in the Appendix A. Importantly, we find clearly positive dependencies among all wind power profiles (see Appendix A.4 for a detailed analysis).

Regarding the level of detail in our analysis, we would ideally opt for a representation of each individual supplier in the system. However, for large systems with many interdependent suppliers, this would quickly involve impractical data requirements and calculation efforts. This is mainly due to the fact that each of the $2^n - 1$ possible coalitions needs to be calculated. Hence, some administrative division may provide a satisfactory level of disaggregation while still being manageable.[16] In our empirical example, we will hence aggregate wind power on the federal state level.[17]

6.2.3. Results

Reliability of Germany's dispatchable power fleet

To determine the cumulative distribution function F_C describing the availability distribution of Germany's dispatchable power supply fleet, we assume that each supplier is either fully available or not,[18] and that individual failure probabilities are independent. Based on these assumptions, F_C

[15]Alternative economic approaches would try to estimate efficient reliability levels from the value of lost load (VOLL) and costs of maintaining a certain level of supply chain reliability (Telson (1975)). A thorough discussion would clearly be beyond the scope of this paper, such that the interested reader is referred to Stoft (2002) for the necessary calculations, and attempts to quantify the VOLL, e.g., by Anderson and Taylor (1986) for Sweden or by Growitsch et al. (2014) for Germany. Noticeably, due to the fact that data requirements and estimation procedures are far from being straightforward, it is not surprising that rules of thumb and common practice, such as the 1-day-in-10-years, are often applied.

[16]The same (relative) Shapley value would then apply to all units within that area.

[17]Germany consists of 16 federal states: Baden-Württemberg (BW), Bayern (BY), Berlin (BE), Brandenburg (BB), Bremen (HB), Hamburg (HH), Hessen (HE), Mecklenburg-Vorpommern (MV), Niedersachsen (NI), Nordrhein-Westfalen (NW), Rheinland-Pfalz (RP), Saarland (SL), Sachsen (SN), Sachsen-Anhalt (ST), Schleswig-Holstein (SH), and Thüringen (TH).

[18]I.e., no partial failures are taken into account.

can be derived via convolution of the individual suppliers' failure probabilities. We implemented the algorithm developed in Hasche et al. (2011) which proved to be fast enough to calculate the cdf for our supply chain (consisting of nearly 900 dispatchable power supply units) in less than a minute on a standard laptop. The resulting (complementary) cumulative distribution function is shown in Figure 1, together with a histogram of load levels. Load levels are for the most part far left of the critical range of around 90 GW where the complementary cdf begins to drop sharply.

From Equation (33) and our data, we obtain a basically negligible risk of supply shortages of $R^{\mathcal{S}} = 1 - 1.15e^{-12}$ hours/year.[19] Hence, installed capacities appear to be largely sufficient to reliably cover today's load profile.

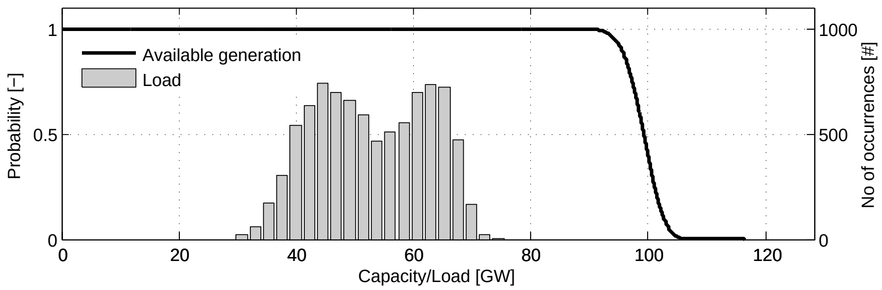

Figure 1: Complementary cdf of Germany's dispatchable power plants together with a histogram of load levels

Having previously mentioned the often applied 1-day-in-10-years target, we find that a 30% increase in current (2013) load levels could be sustained in order to reach that threshold, again indicating large amounts of over-capacity in the German system.

The contribution of wind power to reliability of supply

We find the joint contribution of Germany's aggregated wind power to reliability of supply $v(N)$ to be 3376 MW, corresponding to $\tilde{v}(N) = 9.9\%$ relative to the installed capacity of 34.02 GW.

To provide empirical evidence for the decreasing returns to scale found in Corollary 1, we scale today's wind power capacity by factors of 0 to 3, while assuming unchanged characteristics of load and dispatchable power (Figure 2, left hand side). As expected, the contribution decreases monotonically along a convex function. To empirically confirm gains of diversification (Corollary 2), we first calculate the contribution of wind capacities being installed in two states (S_1 and S_2) separately, ranging from 100 to 900 MW. We take Sachsen (SN) and Thüringen (TH) as an example.[20] We then calculate the joint contribution of a diversified portfolio of the same aggregated

[19]Note that $R^{\mathcal{S}}$ is a risk measure, not implying that a certain number of load shedding events effectively occurs. Hence, the figures presented here and in the following should not be confused with realized statistical numbers, such as the Average System Interruption Duration Index (ASIDI).

[20]These two states have a similar average availability of wind power and a correlation coefficient of 0.59.

capacity being installed in both the states (capacity split half-half, i.e., $v(S_{1,\frac{1}{2}} \cup S_{2,\frac{1}{2}})$). Whereas the joint contribution of the diversified portfolio lies in between the separate states for small capacities, it clearly yields higher contributions for increasing penetration levels. Especially, the rate at which the diversified contribution drops is significantly smaller.

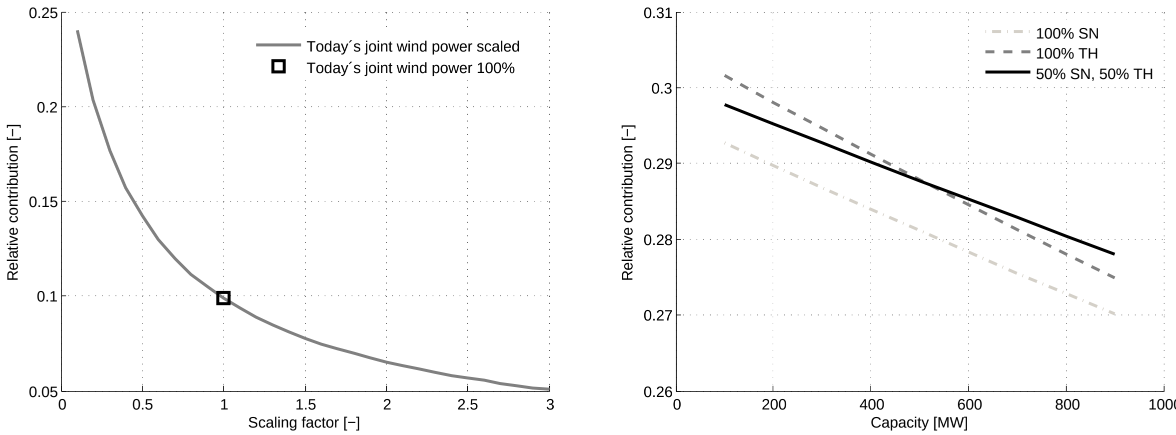

Figure 2: Decreasing returns to scale (left); Gains of diversification (right)

The following Table 1 presents today's installed wind power capacities per state, along with the individual absolute ($v(S_i)$) and relative contributions ($\tilde{v}(S_i)$). Individual relative contributions vary significantly, ranging from 13.9 to 31.3%. Summing up the individual contributions would clearly yield a false joint contribution of 19.6%. Compared to the previously determined consistent joint contribution of 9.9%, this is a huge overestimation that would be induced by neglecting the positive interdependencies between the resource availability at different locations. This empirical finding underlines the importance of the non-additivity property, as stated in Corollary 3.

State	Installed Capacity [MW]	Absolute Contribution [MW]	Relative Contribution
BW	624	156	24.9%
BY	1066	158	14.8%
BE	2	1	31.3%
BB	5233	1013	19.4%
HB	114	34	30.0%
HH	55	16	28.1%
HE	961	145	15.1%
MV	2301	588	25.6%
NI	7676	1392	18.1%
NW	3473	483	13.9%
RP	2366	395	16.7%
SL	223	44	19.9%
SN	1055	281	26.6%
ST	4093	955	23.3%
SH	3683	723	19.6%
TH	1097	295	26.9%

Table 1: Installed capacity, absolute and relative contributions calculated for each state individually

Non-additivity

Empirical evidence shows that our problem is largely subadditive and non-convex, and that the core is empty. The following Figure 3 is meant to illustrate the subadditive nature of our problem from two perspectives. The left hand side shows the coalition's joint contribution vs. its size (in terms of coalition members) for all 65535 possible coalitions,[21] clearly indicating an inverse relation. Similarly, the right hand side shows the error that would be induced by neglecting the (positive) interdependencies by plotting each possible coalition's sum of individual contributions vs. its consistent joint contribution.[22] Whereas superadditivity would require all points to be above the perfect additivity line, we find that virtually all points lie well below, with the error substantially increasing for larger individual contributions (essentially due to decreasing returns to scale, i.e., in line with Corollary 1). The results of both figures are driven by the positive interdependencies among the 16 states that are illustrated in Figure A.4.

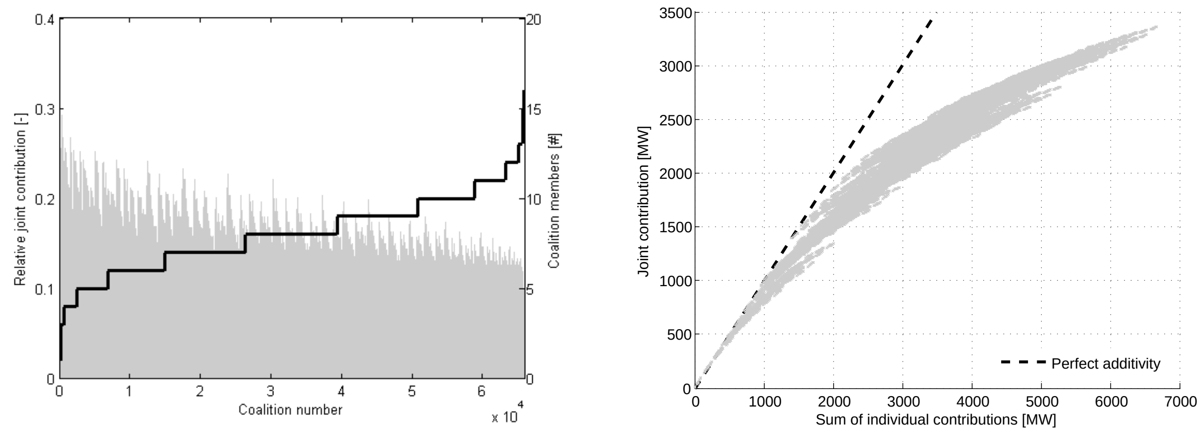

Figure 3: Relative joint contribution vs. size of coalition (left); Sum of individual contributions vs. joint contribution (right)

Prequalification according to the Shapley value

Figure 4 presents the relative Shapley value $\tilde{\Phi}_i(v(N))$, i.e. the payoffs relative to installed capacities allocated to each of the 16 states (squares). To put these values into perspective, they are presented in combination with the suppliers' average availability (μ_i) as well as with their separately calculated individual contribution ($\tilde{v}(S_i)$). We find that the individual contribution as well as the Shapley value tends – as expected – to decrease with average availability. However, the Shapley values also show pronounced deviations from the joint contribution (9.9%), as well as from the ordering of the individual contributions and average availabilities. For instance, Niedersachsen (NI) and Sachsen-Anhalt (ST) both have high average availabilities, but also large installed capacities as

[21]For a game with 16 players, the number of possible coalitions is $2^{16} - 1 = 65535$.

[22]The most exterior point, i.e. the grand coalition of all states, has already been discussed in the previous paragraph.

well as pronounced positive dependencies with other states – and hence comparatively low Shapley values. Overall, the Shapley values range from 15.1% for Baden-Württemberg (BW) and Sachsen (SN) to 5.7% for Bavaria (BY).

Investigating the types of externalities emerging from the problem at hand and relating the Shapley value to the possibility of emerging beneficial coalitions, we find that 50337 out of 65535 possible coalitions would be able to block this allocation, due to the subadditivity found above. Hence, it seems to be reasonable to transfer the process of prequalification to an independent central authority to avoid competitively unstable situations (Telser (1994)).

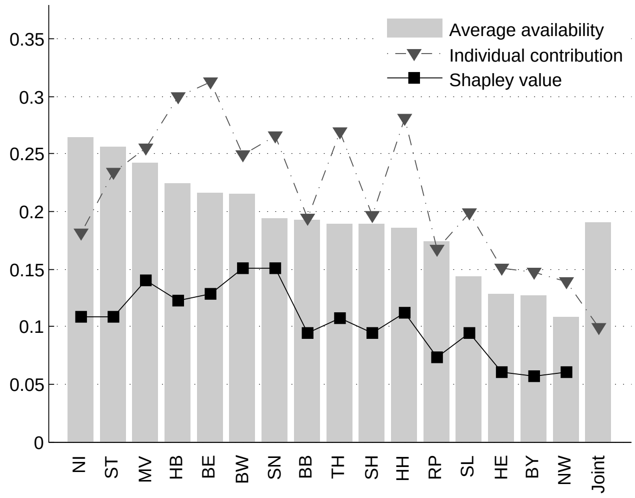

Figure 4: Average availabilities, individual contributions and Shapley values (all relative)

In practice, it will be most plausible to pay power supply units according to a weighted production- and reliability-related preference (i.e., if existing markets for energy are or were to be complemented by a capacity mechanism). Figure 5 presents this tradeoff, showing that states perform strikingly different on both dimensions. Separating the field into quadrants, we find that Baden-Württemberg (BW) and Sachsen (SN) perform particularly well for the case of more preference given to reliability, Niedersachsen (NI) and Sachsen-Anhalt (ST) for the case of more preference on production volumes, and Mecklenburg-Vorpommern (MV) for an equal balance. Nordrhein-Westfalen (NW), Bavaria (BY) and Hessen (HE) perform poorly with respect to both properties. Moreover, it should be noticed that even though a general positive trend can be found, the tradeoff is widely scattered around a straight line. This property would offer the possibility to effectively incentivize investments that contribute much better to reliability than purely production-based decisions by means of appropriate payoffs reflecting (weighted) preferences for production volumes *and* reliability.

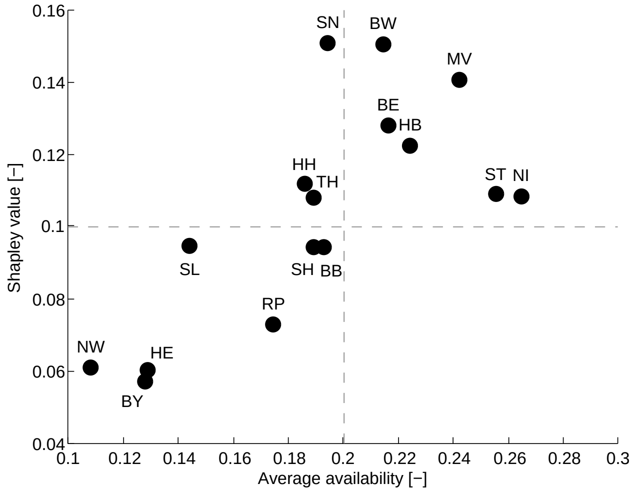

Figure 5: Average availability vs. Shapley value

7. Conclusions

Supply chain reliability is a timely and relevant subject that has been studied intensively in the academic literature. However, the specific role of interdependent suppliers for supply chain reliability has so far been disregarded. To fill this gap, we have thoroughly investigated the issue based on statistical and economic analyses. Especially, we have derived an analytical solution for the contribution of individual suppliers to reliability as well as strategies for the assessment and planning of reliability in supply chains that account for the statistical properties of the problem. Practical applicability and relevance has been demonstrated with a numerical example based on wind energy in the German power system, thereby confirming our analytical findings.

Overall, our concepts and findings may be used to design and manage the reliability of supply chains more efficiently. Especially, integrated supply chains may benefit from a comprehensive optimization, while the Shapley value provides a suitable way to incentivize external suppliers. In power systems, the approach may improve the design of capacity mechanisms, thus enabling an efficient procurement of reliability from a large number of interdependent suppliers. This example appears to be particularly relevant as our analysis has revealed substantial shortcomings in existing designs that can be overcome with our suggested approach.

Our analysis could be extended in several directions: It would be interesting to investigate optimal mechanisms to ensure reliability in supply chains under private information (e.g., regarding the suppliers' costs). One could also consider the role of capacity-constrained transportation networks (which would in fact be highly relevant for the specific case of power systems), or the effect of an endogenous demand-side response on reliability. Lastly, our approach could be applied to other supply chains incorporating stochastic suppliers and demand, such as homogeneous food products or reliable customer services.

References

Agentur für erneuerbare Energien, 2014. Yearly power generation from wind energy per federal state. URL http://www.foederal-erneuerbar.de/uebersicht/kategorie/wind/bundeslaender/BW|BY|B|BB|HB|HH|HE|MV|NI|NRW|RLP|SL|SN|ST|SH|TH|D

Amelin, M., 2009. Comparison of capacity credit calculation methods for conventional power plants and wind power. IEEE Transactions on Power Systems 24 (2), 685–691.

Anderson, R., Taylor, L., 1986. The social cost of unsupplied electricity: A critical review. Energy Economics 8 (3), 139 – 146.

Babich, V., Burnetas, A. N., Ritchken, P. H., 2007. Competition and diversification effects in supply chains with supplier default risk. Manufacturing & Service Operations Management 9 (2), 123–146.

Bazaraa, M. S., Sherali, H. D., Shetty, C. M., 2006. Nonlinear Programming - Theory and Algorithms. John Wiley & Sons.

Billinton, R., 1970. Power system reliability evaluation. Taylor & Francis.

Billinton, R., Allan, R. N., 1996. Reliability evaluation of power systems, 2nd ed. New York: Plenum Publishing Corporation.

Brandenburger, A., Stuart, H., 2007. Biform games. Management Science 53 (4), 537–549.

Bundesnetzagentur, 2014. Kraftwerksliste Bundesnetzagentur (Bundesweit; alle Netz- und Umspannebenen); Stand 16.07.2014. URL http://www.bundesnetzagentur.de/cln_1432/EN/Areas/Energy/Companies/SpecialTopics/PowerPlantList/PubliPowerPlantList_node.html

Bundesverband WindEnergie, 2014. Yearly wind power capacity per federal state. URL https://www.wind-energie.de/themen/statistiken/bundeslaender

Cambridge Energy Research Associates, 2001. Short circuit: will the California energy crisis derail the state's economy? The UCLA Anderson forecast for the nation and California. Los Angeles, California.

Council of European Energy Regulators, 2014. Recommendations for the assessment of electricity generation adequacy.

Cramton, P., Ockenfels, A., Stoft, S., 2013. Capacity market fundamentals. Economics of Energy & Environmental Policy 2 (2), 27–46.

Dada, M., Petruzzi, N. C., Schwarz, L. B., 2007. A newsvendor's procurement problem when suppliers are unreliable. Manufacturing & Service Operations Management 9 (1), 9–32.

dena, 2010. Kurzanalyse der Kraftwerksplanung in Deutschland bis 2020 (Aktualisierung).

Dragoon, K., Dvortsov, V., 2006. Z-method for power system resource adequacy applications. Power Systems, IEEE Transactions on 21 (2), 982–988.

DWD, 2014. Hourly mean values of wind speeds. URL ftp://ftp-cdc.dwd.de/pub/CDC/observations_germany/climate/hourly/wind/historical/

Elberg, C., Hagspiel, S., 2015. Spatial dependencies of wind power and interrelations with spot price dynamics. European Journal of Operational Research 241 (1), 260–272.

ENTSO-E, 2014. Hourly load levels for european countries. URL https://www.entsoe.eu/resources/data-portal/consumption/

Firtin, E., Güler, Ö., Akdag, S. A., 2011. Investigation of wind shear coefficients and their effect on electrical energy generation. Applied Energy 88 (11), 4097–4105.

Floudas, C. A., Visweswaran, V., 1995. Quadratic optimization. In: Handbook of global optimization. Springer, pp. 217–269.

Gallo, G., Hammer, P. L., Simeone, B., 1980. Quadratic knapsack problems. In: Combinatorial Optimization. Springer, pp. 132–149.

Garver, L. L., 1966. Effective load carrying capability of generating units. IEEE Transactions on Power Appartus and Systems 85 (8), 910–919.

Growitsch, C., Malischek, R., Nick, S., Wetzel, H., 2014. The costs of power interruptions in germany: A regional and sectoral analysis. German Economic Review.

Gürler, Ü., Parlar, M., 1997. An inventory problem with two randomly available suppliers. Operations Research 45 (6), 904–918.

Hart, S., Mas-Colell, A., 1989. Potential, value, and consistency. Econometrica: Journal of the Econometric Society, 589–614.

Hasche, B., Keane, A., O'Malley, M., 2011. Capacity value of wind power, calculation, and data requirements: the irish power system case. IEEE Transactions on Power Systems 26 (1), 420–430.

Jorion, P., 2007. Value at risk: the new benchmark for managing financial risk. Vol. 3. McGraw-Hill New York.

Joskow, P. L., 2008. Capacity payments in imperfect electricity markets: Need and design. Utilities Policy 16 (3), 159–170.

Keane, A., Milligan, M., Dent, C., Hasche, B., D'Annunzio, C., Dragoon, K., Holttinen, H., Samaan, N., Soder, L., O'Malley, M., 2011. Capacity value of wind power. IEEE Transactions on Power Systems 26 (2), 564–572.

Li, J., Wang, S., Cheng, T. E., 2010. Competition and cooperation in a single-retailer two-supplier supply chain with supply disruption. International Journal of Production Economics 124 (1), 137–150.

Li, Z., Xu, S. H., Hayya, J., 2004. A periodic-review inventory system with supply interruptions. Probability in the Engineering and Informational Sciences 18 (01), 33–53.

Masih-Tehrani, B., Xu, S. H., Kumara, S., Li, H., 2011. A single-period analysis of a two-echelon inventory system with dependent supply uncertainty. Transportation Research Part B: Methodological 45 (8), 1128–1151.

Minner, S., 2003. Multiple-supplier inventory models in supply chain management: A review. International Journal of Production Economics 81, 265–279.

Nordex, 2007. Nordex s70/s77.
URL `http://www.nordex-online.com/fileadmin/MEDIA/Produktinfos/Nordex_S70-S77_D.pdf`

Parlar, M., Perry, D., 1996. Inventory models of future supply uncertainty with single and multiple suppliers. Naval Research Logistics (NRL) 43 (2), 191–210.

Pisinger, D., 2007. The quadratic knapsack problem - a survey. Discrete applied mathematics 155 (5), 623–648.

Rader, D. J., Woeginger, G. J., 2002. The quadratic 0–1 knapsack problem with series–parallel support. Operations Research Letters 30 (3), 159–166.

Shapley, L., 1953. A value for n-person games. In: Kuhn, H., Tucker, W. (Eds.), Annals of Mathematical Studies. Vol. 28. Princeton University Press, Ch. Contributions to the Theory of Games II.

Snyder, L. V., Atan, Z., Peng, P., Rong, Y., Schmitt, A. J., Sinsoysal, B., 2014. OR/MS models for supply chain disruptions: A review. Available at SSRN 1689882.

Stoft, S., June 2002. Power System Economics: Designing Markets for Electricity. Wiley-IEEE Press.

Tang, C. S., 2006. Perspectives in supply chain risk management. International Journal of Production Economics 103 (2), 451–488.

Tasche, D., Tibiletti, L., 2003. A shortcut to sign incremental value at risk for risk allocation. The Journal of Risk Finance 4 (2), 43–46.

Telser, L. G., 1994. The usefulness of core theory in economics. The Journal of Economic Perspectives 8 (2), 151–164.

Telson, M. L., 1975. The economics of alternative levels of reliability for electric power generation systems. The Bell Journal of Economics 6 (2), 679–694.

Tomlin, B., 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. Management Science 52 (5), 639–657.

VGB and Eurelectric, 2012. Availability of Thermal Power Plants 2002 - 2011, Edition 2012.

Wagner, S. M., Bode, C., Koziol, P., 2009. Supplier default dependencies: Empirical evidence from the automotive

industry. European Journal of Operational Research 199 (1), 150–161.
Wang, Z., 2002. The properties of incremental var in monte carlo simulations. The Journal of Risk Finance 3 (3), 14–23.
Weare, C., 2003. The California Electricity Crisis: Causes and Policy Options. Public Policy Institute of California.
Zachary, S., Dent, C. J., 2011. Probability theory of capacity value of additional generation. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 226 (1), 33–43.

Appendix A. Data and preparatory calculations

Appendix A.1. Installed capacities and availability factors

For information about currently installed power supply units, we use the List of Power Plants prepared and provided by the Federal Network Agency (Bundesnetzagentur (2014)). It lists all existing units in Germany with a net nominal electricity capacity of at least 10 MW.[23] Moreover, supply facilities of less than 10 MW are also included on an aggregated basis grouped by energy source. Extracted net nominal capacities by fuel type are depicted in Table A.1. As for wind power, 34.02 GW were installed by mid 2014, distributed among the 16 federal states as shown in Figure A.1.

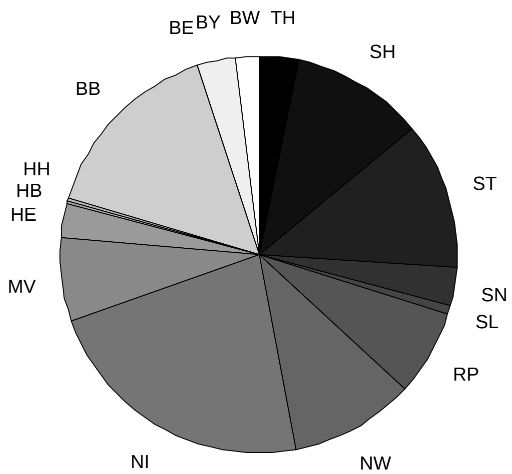

Figure A.1: Distribution of installed wind power capacities (34.02GW total)

Regarding the suppliers' availabilities, we assume fuel-type specific factors according to historical observations, taken from VGB and Eurelectric (2012), and complemented with dena (2010) for Hydro, Geothermal and Biomass, as reported in Table A.1. For PV, we assume an availability factor of 0%.[24]

[23]In addition, it also comprises capacities directly feeding into the German grid from Luxembourg, Switzerland and Austria (which we assume to contribute to Germany's generation adequacy)
[24]This is a conservative estimate, however, consistent with the observation that highest load hours occur in the late evening during winter time.

Fuel type	Availability	Capacity [GW]
Biomass	88.0%	6.38
Coal	83.9%	27.73
Gas	88.3%	25.42
Geothermal	90.0%	0.03
Hydro (pump) storage	90.0%	10.63
Hydro run-of-river	40.0%	3.92
Lignite	85.3%	20.95
Nuclear	83.3%	12.07
Oil	89.2%	4.14
Others (Waste, Landfill gas, etc.)	90.0%	5.32
PV	0.0%	37.45
Wind	*to be calculated*	34.02

Table A.1: Availability factors and installed capacities per fuel type

Appendix A.2. Wind speed data and wind to power conversion

As we want to focus on the supply side uncertainty of wind power in our empirical example, coverage and resolution of the wind data are crucial for obtaining reliable results. Consequently, we use hourly wind speed data of 32 years (1982-2013) to cover a broad range of possible wind patterns in Germany, provided by the national climate monitoring of the German Weather Service (DWD (2014)).[25] We select one representative location per federal state according to the agglomeration of wind turbines within each state.[26]

The conversion of wind speed to electrical power output is described by a turbine-specific power curves. As a representative power curve, we use the Nordex S77 turbine with a hub height of 77 meters and a rated power of 1.5kW (Nordex (2007)). Wind speed to power conversion is implemented via lookup-tables with linear interpolation.

To validate this relatively simple model for generating wind power profiles, we compare statistical profiles and volumes with our synthetic model-generated data. Figure A.2 compares statistical yearly production per federal state (available from Agentur für erneuerbare Energien (2014) for the years 2001-2012, except for 2007) against our modeling results (based on historically installed wind capacities taken from Bundesverband WindEnergie (2014) and wind speeds from the corresponding years). As can be seen, our model overestimates production for the first years, whereas satisfactory conformity is reached for more recent periods. This is probably due to improvements in turbine technologies over the years (remember that we have applied the power curve of a relatively modern turbine).

[25]In case of missing data, empty entries are replaced by interpolations based on the previous and next available value if the empty space is not exceeding 12 hours. If the gap is longer, entries are replaced by data of the same station and same hours of the previous year. As measurements are taken a couple of meters above ground only, wind speeds are scaled to the wind turbines' hub height assuming a power law: $v_{h_1} = v_{h_0}(h_1/h_0)^a$, where h_0 is the measurement height, h_1 the height of interest and α the shear exponent. According to Firtin et al. (2011), a is assumed to be 0.14.

[26]The list of selected observatories is presented in the following Table A.2.

State	DWD-ID	Observatory name
BW	4887	Stötten
BY	5705	Würzburg
BE	3987	Potsdam
BB	164	Angermünde
HB	691	Bremen
HH	1975	Hamburg-Fuhlsbüttel
HE	1420	Frankfurt
MV	4271	Rostock
NI	891	Cuxhaven
NW	2483	Kahler Asten
RP	2385	Idar-Oberstein
SL	4336	Saarbrücken
SN	1048	Dresden
ST	1957	Halle-Kröllwitz
SH	4466	Schleswig
TH	1270	Erfurt-Weimar

Table A.2: Selected DWD observatories

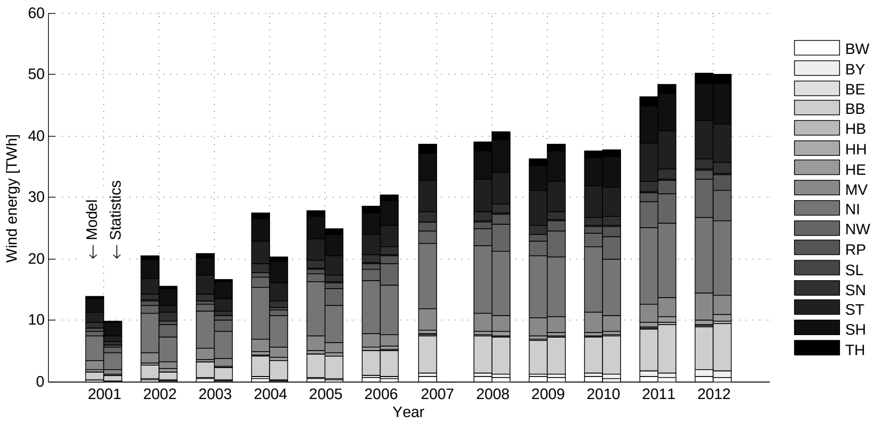

Figure A.2: Yearly wind power production per federal state (model: left, statistics: right)

Another step of validation has been carried out for the hourly profiles, based on a comparison of the distributions of the model-based data with historical production profiles (available for the years 2010-2013) by means of QQ-plots (Figure A.3). Distributions are found to be very similar for those years, however, with slightly decreasing conformity for upper quantiles. Conducting a simple regression analysis yields an R^2 of 0.80, 0.81, 0.82 and 0.81 for the years 2010-2013. For completeness, summary statistics of the obtained profiles (based on 2014 wind capacities) are also provided in Table A.3.

Appendix A.3. Load profiles

Germany's load levels are reported on an hourly basis by ENTSO-E (2014), representing the hourly average active power absorbed by all installations connected to the transmission or distribution network. Instead of using multiple years of load data, we restrict our attention to the most

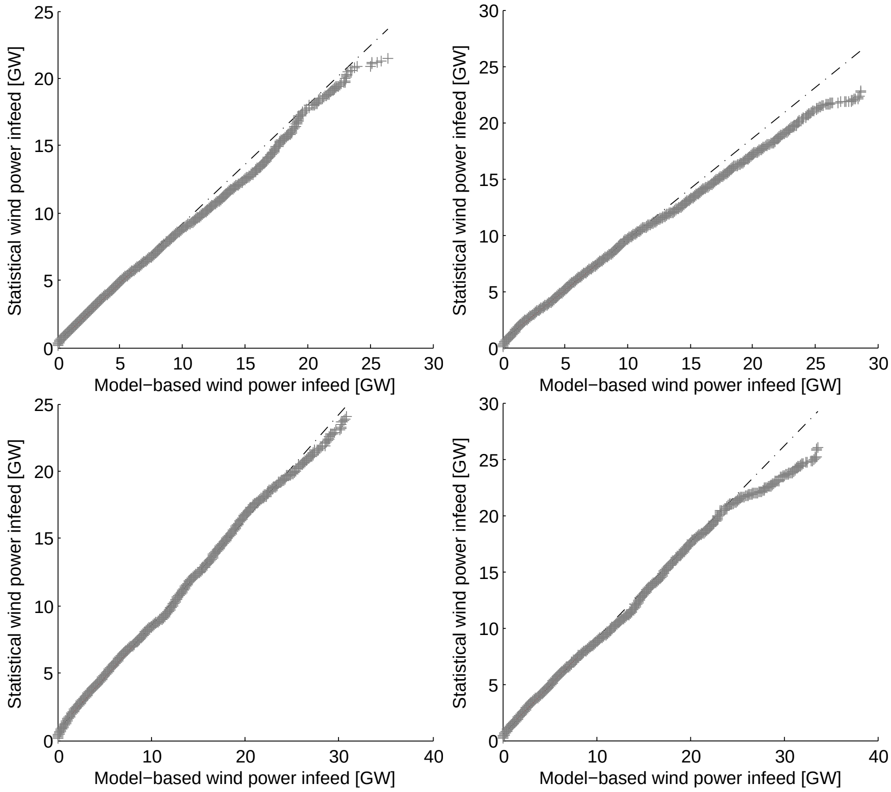

Figure A.3: QQ-plots of model-based and historical production profiles, for the years 2010-2013

Profile	Capacity	Min	Max	Mean	Std
Load Germany (2013)	n.a.	29.55	75.623	52.892	9.633
Wind Power BW (1982-2013)	0.624	0	0.624	0.134	0.183
Wind Power BY (1982-2013)	1.066	0	1.066	0.136	0.251
Wind Power BE (1982-2013)	0.002	0	0.002	0.000	0.001
Wind Power BB (1982-2013)	5.233	0	5.233	1.010	1.406
Wind Power HB (1982-2013)	0.114	0	0.114	0.026	0.032
Wind Power HH (1982-2013)	0.055	0	0.055	0.010	0.014
Wind Power HE (1982-2013)	0.961	0	0.961	0.124	0.215
Wind Power MV (1982-2013)	2.301	0	2.301	0.557	0.731
Wind Power NI (1982-2013)	7.676	0	7.676	2.033	2.398
Wind Power NW (1982-2013)	3.473	0	3.473	0.375	0.723
Wind Power RP (1982-2013)	2.366	0	2.366	0.413	0.669
Wind Power SL (1982-2013)	0.223	0	0.223	0.032	0.051
Wind Power SN (1982-2013)	1.055	0	1.055	0.205	0.287
Wind Power ST (1982-2013)	4.093	0	4.093	1.047	1.326
Wind Power SH (1982-2013)	3.683	0	3.683	0.696	0.932
Wind Power TH (1982-2013)	1.097	0	1.097	0.207	0.308

Table A.3: Capacities and summary statistics of load and wind power profiles in [GW]

recent year 2013 in order to focus on the supply side uncertainty. We hence refer to reliability under current load levels and profiles. Summary statistics of the load profile are comprised in Table A.3.

Appendix A.4. Correlation analysis

As has been noticed several times during the theoretical analysis in Sections 3 and 5, covariance among the power production of the different locations crucially impacts the properties and results of the problem. In order to get an impression of the dependencies characterizing wind power and load in Germany, Figure A.4 shows the matrix of linear correlation coefficients ρ.

As can be seen, ρ among the wind power profiles is in a range of $[0.10, 0.77]$, with a mean of 0.48 (excluding diagonal values). Correlations between wind power and load are in a range of $[0.06, 0.17]$, with a mean of 0.13. Hence, all dependencies are clearly positive.[27]

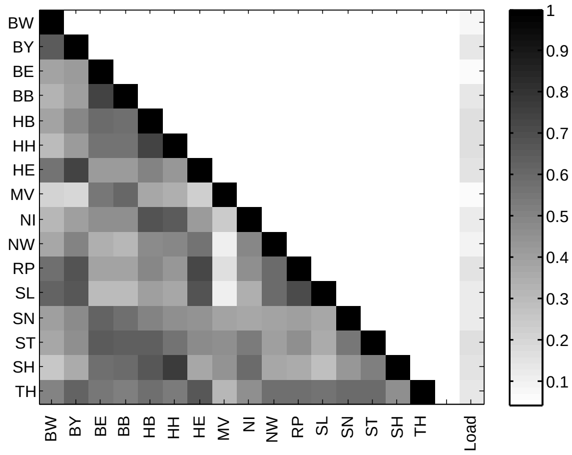

Figure A.4: Correlation matrix of wind power profiles

[27] As reliability of supply is particularly relevant during tight capacities, we recalculate the same numbers for the data corresponding to the 5% highest load hours, resulting in a range of $[0.15, 0.79]$ and a mean of 0.51 for correlations among wind power, and $[-0.02, 0.03]$ and a mean value of 0.00 for correlations between wind power and load. Hence, wind power dependencies are even slightly more pronounced during high-load-hours, whereas wind power and load are independent.